ciety and in the Athenæum Club, where it delighted Sir Wm. Thomson, now Lord Kelvin, and led to the extraordinary lecture *On Recent Discoveries in Mechanical Conversion of Motion*, delivered by Sylvester before the Royal Institution on January 23, 1874. This in turn led to Kempe's remarkable development of the subject, and to Hart's discovery of a five-bar linkage which does the same work as Peaucellier's of seven.

Henceforth Peaucellier's Cell and Hart's Contraparallelogram will take their place in our text-books of geometry, and straight lines can be drawn without begging the question by assuming first a straight edge or ruler as does Euclid.

Thus Kempe's charming book, '*How to Draw a Straight Line*,' is a direct outcome of Tchébychev's sketch for Sylvester. As might perhaps have been expected, the immortal Lobachévsky found in his compatriot a devoted admirer. Not only was Tchébychev an active member of the committee of the Lobachévsky fund, but he took the deepest interest in all connected with the spread of the profound ideas typified in the non-Euclidean geometry. Knowing this, Vasiliev in his last letter asked that a copy of my translation of his address on Lobachévsky be forwarded to the great man. His active participation in scientific assemblies is also worthy of note ; for example, at the 'Congrès de l' association française pour l'avancement des sciences, à Lyon,' he read two interesting papers, *Sur les valeurs limites des intégrales*, and *Sur les quadratures*, afterwards published in *Liouville's Journal*.

GEORGE BRUCE HALSTED.
UNIVERSITY OF TEXAS.

SCIENTIFIC LITERATURE.

Les Oscillations Électriques. H. POINCARÉ.
(CONCLUDED.)

Propagation of Electrical Oscillations Through Air.—The velocity of propagation of electromagnetic induction through dielectrics offered the first experimental test of superiority of the Faraday-Maxwell theory over the older theories. According to these that velocity should be infinite ; according to the Faraday-Maxwell view of electromagnetic phenomena it should be the same as that of light. Poincaré reviews carefully all the experimental evidences bearing upon this point. Hertz's experiments in Carlsruhe are first discussed and his early failures in arriving at a satisfactory result are pointed out. Two methods employed in these measurements by Hertz at Carlsruhe and at Bonn are described briefly. One of these consisted in measuring by means of a resonator the difference of phase between two waves sent forth by the same oscillator, one wave along a conducting wire and the other through the dielectric in the vicinity of the wire. The other method consisted in measuring what Hertz considered the wave length of stationary electric waves in air formed by the interference between the direct waves sent forth by an oscillator and the waves reflected by a large flat mirror consisting of a metal sheet 2 meters wide and 4 meters high. In all these experiments the velocity of propagation along the wire seemed to come out considerably different from and generally less than that in air. But the methods were open to several criticisms. In the first place, the hall in which these experiments were carried out was too small for the wave lengths employed ; secondly, the influence of the waves reflected from the walls was entirely neglected ; thirdly, the dimensions of the reflecting mirror were not large enough in comparison to the wave length to prevent errors of observation due to the misleading influence of diffraction phenomena. All these objections were in a measure overcome in the earliest experiments of Sarasin and de la Rive (C. R. t. CX. p. 72). In these experiments the methods of Hertz were employed, but they were performed in a large hall, with a large

mirror and with smaller resonators. The results improved with the increase of the dimensions of the mirror and the diminution of the size of the exploring resonators. In a subsequent series of experiments (C. R. CXX., p. 688) carried out in a very large hall with a mirror 8 meters high and 16 meters wide and employing circular resonators of 50 and 75 centimeters in diameter these investigators obtained completely satisfactory results, proving beyond all reasonable doubt that the velocity of propagation of electromagnetic waves through dielectrics is the same as along conducting wires and equal to the velocity of light. The sources of error in Hertz's experiments were clearly demonstrated by these experiments, for no matter how large were the hall and the mirror a sufficient increase in the dimensions of the exploring resonators would always give misleading results, similar to those obtained by Hertz.

But among the many encouraging results obtained by Sarasin and de la Rive there is one result which causes much anxiety to the mathematical physicist. It is the serious disagreement between the theoretically calculated period of the resonator and that determined experimentally by the illustrious physicists of Geneva. In an exceedingly interesting mathematical discussion of the functions of the resonator Poincaré shows that the wave length of the fundamental vibration can differ but little from twice the circumference of the resonator, whereas Sarasin and de la Rive found it to be equal to eight times the diameter. The cause of this disagreement must be explained by the theory, but how? Poincaré gives no definite answer to this question. Many valuable suggestions are thrown out, however, and the subject is then dismissed after showing by a reference to Blondlot's and Bjerkness' experiments that the theory of the resonator just given is correct in its main features. No other theory of the resonator has been given since that given by Hertz, and Poincaré's discussion contains many valuable additions to the rough outline of the subject sketched out by Hertz. In this connection the reviewer ventures to refer to a paper by Professor P. Drude (*Zum Studium des Electrischen Resonators*, Wied. Ann. Nov. 1894).

Reflection and Absorption of Hertzian Waves.—Resonator and mirror form the essential instruments in every method of studying electrical waves in the dielectric. The phenomena of reflection and absorption of these waves deserve, therefore, careful analysis. To these Poincaré devotes his attention now. The case of orthogonal incidence upon a plane metal mirror is first discussed. It is shown that the penetration of the wave into the metal is inversely proportional to the square root of the product of conductivity and permeability of the metal and directly proportional to the square root of the wave length. For instance, a wave of a periodicity of 50 millions per second, which is the ordinary Hertzian frequency, will be reduced to nearly one-third of its initial intensity at a distance of $\frac{1}{50}$ mm. below the surface of a mirror of copper. The relation, however, which Poincaré obtains between the penetrability of the wave and the wave length, the conductivity, permeability, and specific inductive capacity of the metal does not hold good for frequencies as high as those of light, for on the one hand it gives by approximation a negative value for the specific inductive capacity of all metals, and on the other hand it gives a conductivity 300 to 400 times smaller than that obtained by ordinary resistance measurements. The same relations hold good for oblique reflection. It is interesting to note that if, as Cauchy believes, the fundamental equations of Fresnel (slightly modified) hold good for metallic reflection then a retardation in phase equal to half a period takes place at

the reflecting surface when the electric force of the incident wave is normal to the plane of incidence; no retardation takes place if this electrical force is in the plane of incidence. The extinction of the wave in its passage through the metal develops heat and Poincaré calculates the rate at which the heat is developed by a given current, obtaining the interesting result that it is proportional to the square root of the product of frequency, specific resistance and permeability. The results of these considerations are now compared to experiment. The most important experiments bearing upon this part of the theory are those of Bjerkness (l. c.). A circular resonator having a small plate condenser interposed in place of the spark gap was employed. Between these plates a small aluminum sheet was suspended and measured by its deflection the mean square of the potential difference between the plates. The oscillator was gradually tuned and the resonance effect in the resonator measured by the deflection of the aluminum sheet. Six resonators of the same dimensions but of different material were investigated. The resonance curve of copper was highest, then followed brass, silver, platinum, nickel and iron, in the same order as required by theory. The resonator decrement of iron, for instance, was nine times and that of platinum twice as large as that of copper. To measure the depth of penetration these materials were deposited electrolytically, say iron on a copper resonator, or *vice versa*, and the resonator effect measured for the various thicknesses of the deposit. Results agreeing very fairly with the theory were obtained.

Propagation of Electrical Waves through Dielectrics other than Air.—Another crucial test of the correctness of the Faraday-Maxwell theory is furnished by the well known relation that the specific inductive capacity of a dielectric is equal to the square of its index of refraction. This relation is an immediate inference from the new electromagnetic theory. Since the index of refraction of a substance is equal to the ratio of the velocity of propagation in vacuum to that in the substance it follows that the velocity of propagation of a Hertzian wave in dielectrics having a specific inductive capacity larger than unity should be smaller than in air. This relation was tested by Blondlot in the experiments cited above by immersing both the conducting wire and the resonator in a liquid dielectric and measuring the wave length. Another method based upon the same principle was that employed by Rubens & Arons (Wied. Ann. 40 p. 585). The neutral point of a rectangular resonator was connected directly to one side of the spark-gap of the oscillator. No spark was then observed in the spark-gap of the resonator. If, however, the balance of the resonator was now disturbed by inserting on one side of it a certain length of wire immersed in a dielectric the spark appeared. The balance was again restored by inserting a sufficient length of wire in the other side of the resonator. The ratio of these two lengths of wire measured the ratio of the velocities of propagation in air and in the dielectric.

Another method, first employed by J. J. Thomson (Phil. Mag. 30, p. 129), was based on the relation which exists between the capacity of a plate condenser and the dielectric constant of the insulator separating its plates. The period of an oscillator or resonator will vary with the dielectric between the condenser plates. Thomson measured the period of an oscillator for various dielectrics placed between its condenser plates and calculated from it the specific inductive capacity. Several other electromagnetic methods are described briefly by Poincaré, and then the statical methods, belonging most of them to the pre-Hertzian epoch, are passed in quick review. Finally the experimental results are coördinated

and briefly discussed. In a large number of cases Maxwell's relation is confirmed; but, again, the cases are numerous in which the agreement between theory and experiment is far from satisfactory; this is especially true of dielectrics showing traces of conductivity and large electric absorption, and even more true of electrolytes. This part of Poincaré's work is rather incomplete, probably because it offers fewer opportunities to a mathematical physicist than any other part of Maxwell's electromagnetic theory. The most serious criticism, perhaps, that may be brought against it is its omission of some of the most important investigations on dielectric constants, as, for instance, the investigations of Boltzmann. Again, not a single word is said concerning the influence which the study of the dielectric properties of substances had upon Faraday and Maxwell and how much it had contributed to the formation of their electromagnetic theory.

The reflection of electrical waves from the surface of a dielectric is taken up and it is shown by a reference to analogous phenomena in optics why reflection cannot occur when the thickness of a dielectric plate is small in comparison to the wave length of an electrical wave. Trouton's experiments (Nature, Vol. 39, p. 391) form the basis of this discussion.

The experimental evidence furnished by the study of the reflection of electrical waves is cited which supports the view that the plane of polarization as defined in optics is perpendicular to the direction of the electrical force in the wave-front.

A very interesting experimental investigation published by Klemencic (Wiener Sitzungsber, 19. Feb., 1891) is next described. It treats of wave reflection by dielectrics. The dielectric experimented with was a slab of sulphur 120 cm. long, 80 cm. wide and 7 cm. thick. The wave length employed was 60 cm. A rectilinear oscillator placed in the axis of a cylindrical parabolic mirror furnished the plane waves. The reflected and refracted waves were studied by means of thermoelectric couples attached to rectilinear oscillators placed in the axis of parabolic mirrors similar to the one used in connection with oscillator. There was a reflection at every angle of incidence when the direction of oscillation of the electrical force was perpendicular to the plane of incidence. But when it was parallel to it then there was an angle of incidence at which no reflection occurred. Fresnel's fundamental formulæ, however, were not quite satisfactorily verified. Poincaré ascribes it to the insufficient thickness of the slab. Klemencic found also that the energy of the incident wave was smaller than the sum of the energies of the reflected and refracted wave, a result which he believed to be due to the presence of diffraction.

Conductors in Motion in an Electromagnetic Field.—The last chapter gives the essential features of Hertz's essay: On the fundamental equations of the electromagnetic field for conductors in motion.

Poincaré considers first the *electromotive force* induced in a circuit which is moving through a variable electromagnetic field. He proceeds as follows: Consider a surface formed by the circuit under consideration. Let it move with the circuit. Consider two consecutive positions of this surface, the time of passage from the first to the second position being infinitely short, the velocity of motion being finite. Consider now the space bounded by the initial and the final position of the surface and by the ring-shaped surface whose boundary is the initial and the final position of the circuit. The total magnetic flux through this surface is according to well known relations proportional to the total amount of what Hertz and Poincaré call *true magnetism* included in the bounded surface. The total induced electromotive force being equal to the total

rate of variation of the magnetic flux through the circuit the last relation leads to the following final result: The total electromotive force induced in an infinitely small circuit which moves through a variable electromagnetic field is composed of three parts. First, the electromotive force due to rate or variation of the magnetic flux through the circuit and produced by the time variation of the field itself. Second, the electromotive force due to the rate of variation of the magnetic flux through the circuit produced by the motion of the circuit. The third component of the induced electromotive force can be described as follows: Suppose that permanent magnetic charges are distributed in any way whatsoever throughout the field. There is then a transference of magnetic matter through the moving circuit. We may call it the magnetic convection current, following a suggestion of Hertz (Unters. ueb. d. Ausbr. der el. Kraft, p. 265). This magnetic convection current is equal to the quantity of magnetic matter contained in the volume traced out per unit of time by the moving circuit, and is proportional to the third component of the induced electromotive force. This component does not appear in Maxwell's theory, so that the Hertzian equations seem to be more complete than those of Maxwell.

Poincaré recognizes in this quite a difference between Maxwell's presentation of the electromagnetic theory and that of Hertz; but this difference will evidently exist only if it is proved that a distribution of permanent magnetism, whose induction flux over a closed surface is a constant, different from zero, can exist. The physical meaning of such a distribution is far from being clear, and Poincaré might have well devoted more attention to the elucidation of this perplexing feature of the Hertzian equations. On this point the student will do well to consult Boltzmann (Vorles. über Maxwell's Theorie d. Elec. & d. Lichtes, II. Theil, IX. Vorles.).

The second group of equations refers to the magnetomotive force induced in a circuit which is changing its position with respect to a field of given distribution of electrical force and it is shown that the total magnetomotive force induced in an infinitely small circuit in motion is composed of four components. The first component is proportional to the rate of change of the flux of electric induction which constitutes the conduction current. The second component is proportional to the rate of change of the flux of electric induction which constitutes the displacement current. The third component is proportional to the rate of change of the electric flux due to the motion of the circuit, and the fourth component is proportional to the convection current of permanent electrostatic charges, corresponding to what was called above the convection current of permanent magnetism. There is, however, no difficulty of conceiving a permanent electrification of the dielectric such that the total flux of its induction through a closed surface should be different from zero, and, therefore, the magnetomotive force induced by an electrical convection current is *a priori* evident as soon as the correctness of the fundamental assumptions in the Faraday-Maxwell theory is admitted. There is no difference between this second group of equations and those given by Maxwell.

It is pointed out that the existence of the third component was verified by Rowland's experiments (Pogg. Ann. 158, p. 487), and the existence of the fourth component by the experiments of Roentgen (Wied. Ann. 35, p. 264). The magnetomotive force due to displacement currents was, of course, first pointed out by the experiments of Hertz.

Next follows a beautiful mathematical discussion of the mechanical forces acting upon a body which is moving through an electromagnetic field. The following types

of forces are passed in quick review: 1. An ordinary magnetic force due to the presence of permanent magnetism. 2. Ordinary electrostatic force due to the presence of electrostatic charges. 3. Electromagnetic force consisting of four distinct components. One component is the electromagnetic action of the field upon conduction currents. The second component is the electromagnetic action of the field upon the displacement currents. The third component corresponds to the electromagnetic action of the field upon the currents observed by Rowland and Roentgen. The fourth type of force is that between a variable current and the electrical reactions set up in the field by its variation. All these forces except the last have been observed experimentally. The last one is too feeble to be detected by any of the known experimental methods.

The work is, unfortunately, marred by quite a number of typographical errors. Some of them occur in the midst of important and rather difficult mathematical operations and will undoubtedly be a source of considerable perplexity to the younger students for whom, especially, this work is intended.

The reviewer is of the opinion that he will reëcho the sentiment of every lover of the Faraday-Maxwell electromagnetic theory when he states that this, the latest, contribution of the brilliant French mathematician will be a welcome guide to everyone who wishes to keep in close contact with the latest advances of the electromagnetic theory.

M. I. PUPIN.

COLUMBIA COLLEGE.

The Steam Engine and Other Heat Engines. By J. A. EWING, Professor of Mechanism and Applied Mechanics in the University of Cambridge. Cambridge University Press; New York, Macmillan & Co. 1894. 8vo., pp. xiv + 400. Price, $3.75.

Professor Ewing, in his article on the steam engine in the *Encyclopædia Britannica,* gave good measure to his ability and knowledge of the subject by the production of a treatise in which, for the first time, a systematic and fairly complete discussion was attempted of the theory of the real steam engine, as distinguished from the purely Thermodynamic Theory of the Ideal Heat Engine, which only had previously been presented by writers on that wonderful machine. Clark and Hirn and Iserwood had cleverly shown the wide discrepancy between the ideal and the real engine, and Cotterill had discussed with elegance and clearness the extra thermodynamic losses of the machine; but Ewing brought together, for the first time, and in such form as to make his discussion useful, to theorist and 'practical man' and professional engineer alike, in the study of existing engines and in the attempt to improve upon them by scientifically accurate designing and construction. His article was a condensed, but complete, exposition to its date, of scientific and practical knowledge of the methods of economical production of heat in the boiler, and of the economical thermodynamic utilization of the energy thus made available at the engine, with exact accounts of the various methods of waste of thermal and of dynamic energy. Had its author written nothing else, this article would have sufficed to give him a full share of fame.

His new treatise on the steam engine, now issued in book form, is based upon his earlier discussion, but is entirely rewritten to give it a shape better adapted to its present purpose, and to permit the introduction of new matter. "The endeavor has been, throughout, to make evident the bearing of theory on practical issues." Some space is devoted to experimental work and the discussion of facts and data revealed by it. In so condensed a work it would have been impossible to introduce as complete a study of pure thermodynamics as may be found in